Post graduate Exam (Basic Engineering Sciences)
Branch: Engineering Mathematics (Master 600)

Menofia University **Faculty of Engineering** **Academic Year: 2016-2017** **Department: Basic Eng. Sci.**		**Subject: Integral Equations** **Code: BES 625** **Time Allowed: 3 hours** **Date: 10 / 6 / 2017** **Max Marks: 100**

Answer all the following questions:

Question 1 (50 mark)

1. Classify each of the following integral equations as Volterra or Fredholm integral equation, linear or nonlinear, and homogeneous or nonhomogeneous:

   (a) $u(x) = \cos x + \int_0^{\frac{\pi}{2}} \cos x \; u(t)\, dt$

   (b) $u(x) = 1 + \frac{x}{4} + \int_0^x \frac{1}{x+t} \frac{1}{u(t)}\, dt$

2. Derive an equivalent I.E. to the following initial value problems:

   $$y''(x) + 5\, y'(x) + 6\, y(x) = 0 \;\; , y(0) = 1 \;\; , \; y'(0) = 1$$

3. Using the recursion series method solve the following I.E.

   $$\emptyset(x) = x + \lambda \int_0^1 \emptyset(s)\, ds$$

4. Consider the I.E.

   $$f(x) = g(x) + \lambda \int_0^{\pi} \sin(x - y)\; f(y)\, dy$$

   Find :

   1) the values of (λ) for which it has a unique solution.
   2) the solution in this case.
   3) the resolvent kernel.
   4) the values of (λ) for which the solution is not unique.

Question 2 (50 mark)

1. Solve the following I.E. of the second kind of the convolution type using successive approximation method:

   $$u(x) = f(x) + \lambda \int_0^x e^{x-t}\, u(t)\, dt$$

2. Obtain the solution of the I.E.

   $$u(x) = 1 + 2 \sin x - \int_0^x u(t)\, dt$$

   using the series method

3. Reduce the initial value problem

   $$y''(x) + 4y(x) = \sin x \;\; , y(0) = 0 \, , \; y'(0) = 0$$

   to Volterra integral equation of the second kind and then find its solution.

4. Use the successive approximation to solve the following Fredholm integral equation

   $$u(x) = \sin x + \int_0^{\frac{\pi}{2}} \sin x \cos t \;\; u(t)\, dt$$

Good luck